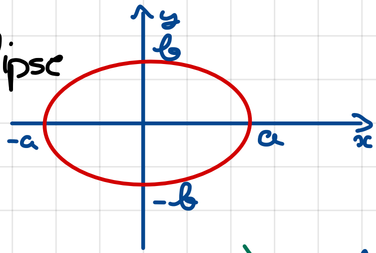
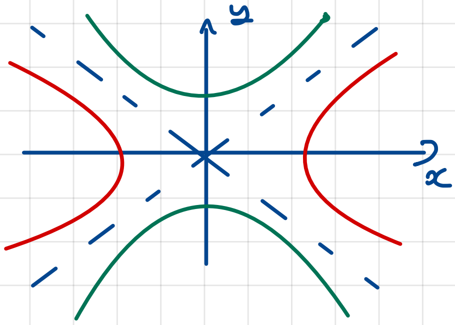
Aula 10

Gráficos: 2 Dimensões

- $y = mx + b \to$ reta
- $y = ax^2 + bx + c \to$ parábola vertical $\nearrow a>0 \to \cup$, $\searrow a<0 \to \cap$
- $x = ay^2 + by + c \to$ parábola horizontal $\nearrow a>0 \to ($, $\searrow a<0 \to)$
- $(x-x_0)^2 + (y-y_0)^2 = r^2 \to$ circunferência de centro (x_0, y_0) e raio r
- $(x-x_0)^2 + (y-y_0)^2 = r^2 \to$ círculo de centro (x_0, y_0) e raio r
- $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \to$ elipse centro $(0,0)$



$$\boxed{\begin{array}{l} \bullet\ \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \\ \bullet\ \dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1 \end{array}} \rightsquigarrow \text{Hipérboles}$$



A vermelho $\to$ 1ª equação

A verde $\to$ 2ª equação

3 Dimensões: Ver ficheiro das quádricas.

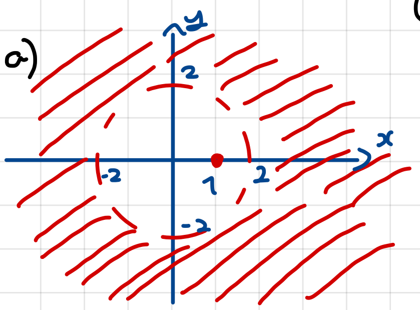
Exercício 1: $n=1$ e $n=3 \to$ T.P.C.

$\boxed{n=2}$ Sejam $X = (x,y)$ e $C = (c_1, c_2)$

Então $\|x - c\| < r \Leftrightarrow \sqrt{(x-c_1)^2 + (y-c_2)^2} < r \Leftrightarrow (x-c_1)^2 + (y-c_2)^2 < r^2$

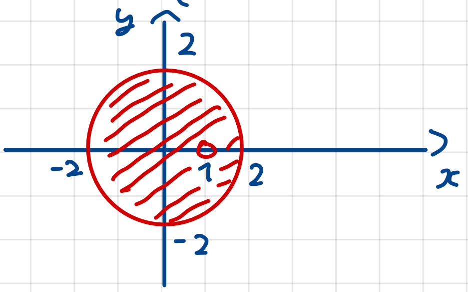
Exercício 2: $D = \{(x,y) \in \mathbb{R}^2 : \underset{\substack{\downarrow \\ r^2 = 4 \Rightarrow r = 2}}{x^2 + y^2 > 4}\} \cup \{(1,0)\}$

a)



b) Complementar de D

$\hookrightarrow D^c = \{(x,y)\in\mathbb{R}^2 : x^2+y^2 \leq 4\} \setminus \{(1,0)\}$



c) $\text{int}(D) = \{(x,y)\in\mathbb{R}^2 : x^2+y^2 > 4\}$

$\text{ext}(D) = \{(x,y)\in\mathbb{R}^2 : x^2+y^2 < 4\} \setminus \{(1,0)\}$

$\text{fr}(D) = \{(x,y)\in\mathbb{R}^2 : x^2+y^2=4\} \cup \{(1,0)\}$

Nota: $\text{int}(D) \cup \text{ext}(D) \cup \text{fr}(D) = \mathbb{R}^2$

$D' = \{(x,y)\in\mathbb{R}^2 : x^2+y^2 \geq 4\}$

Pontos isolados: $\{(1,0)\}$

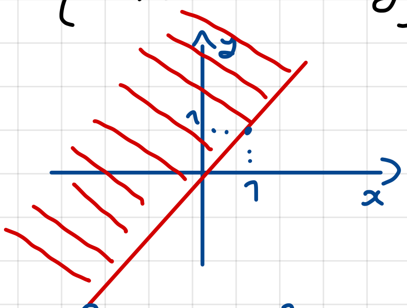
$\overline{D} = D \cup \text{fr}(D) = \{(x,y)\in\mathbb{R}^2 : x^2+y^2 \geq 4\} \cup \{(1,0)\}$

d) D não é aberto pois $D \neq \text{int}(D)$

e) D não é fechado pois $\overline{D} \neq D$

Exercício 3:

a) $D = \{(x,y)\in\mathbb{R}^2 : x \leq y\}$



$\text{int}(D) = \{(x,y)\in\mathbb{R}^2 : x < y\}$

$\text{fr}(D) = \{(x,y)\in\mathbb{R}^2 : x = y\}$

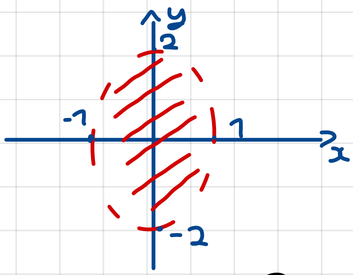
$\overline{D} = D \cup \text{fr}(D) = \{(x,y)\in\mathbb{R}^2 : x \leq y\}$

D é fechado pois $\overline{D} = D$

b) $D = \{(x,y)\in\mathbb{R}^2 : 4x^2+y^2 < 4\}$ $\div 4$

$x^2 + \frac{y^2}{4} < 1 \Leftrightarrow \frac{x^2}{1^2} + \frac{y^2}{2^2} < 1$

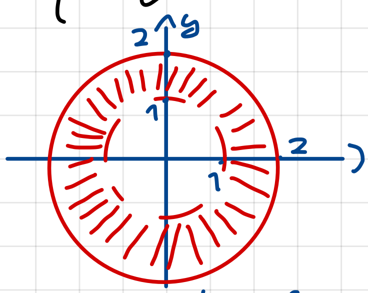
interior de uma elipse



$\text{int}(D) = D \rightarrow D$ é aberto

$\text{fr}(D) = \{(x,y)\in\mathbb{R}^2 : 4x^2+y^2 = 4\}$

c) $D = \{(x,y)\in\mathbb{R}^2 : 1 < x^2+y^2 \leq 4\}$



$\text{int}(D) = \{(x,y)\in\mathbb{R}^2 : 1 < x^2+y^2 < 4\}$

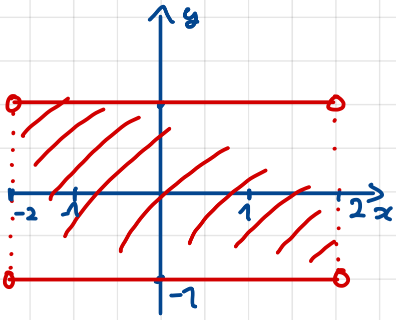
$\text{fr}(D) = \{(x,y)\in\mathbb{R}^2 : x^2+y^2=1 \vee x^2+y^2=4\}$

$\overline{D} = \{(x,y)\in\mathbb{R}^2 : 1 \leq x^2+y^2 \leq 4\}$

$\text{int}(D) \neq D \rightarrow D$ não é aberto

$\overline{D} \neq D \rightarrow D$ não é fechado

d) $D=\{(x,y)\in\mathbb{R}^2: |x|<2 \wedge |y|\le 1\}$



$\text{int}(D)=\{(x,y)\in\mathbb{R}^2: |x|<2 \wedge |y|<1\}$

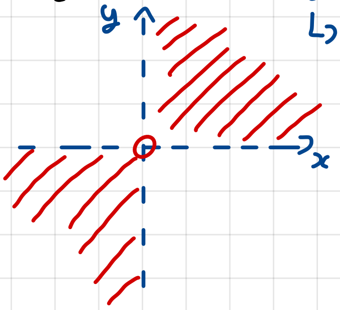
$\text{fr}(D)=\{(x,y)\in\mathbb{R}^2: (|x|=2 \wedge |y|\le 1) \vee (|y|=1 \wedge |x|\le 2)\}$

$\overline{D}=\{(x,y)\in\mathbb{R}^2: |x|\le 2 \wedge |y|\le 1\}$

Não é aberto nem fechado

e) $D=\{(x,y)\in\mathbb{R}^2: \underbrace{xy>0}\}$

$\hookrightarrow \overbrace{(x>0 \wedge y>0)}^{\text{1º quad}} \cup (x<0 \wedge y<0)$

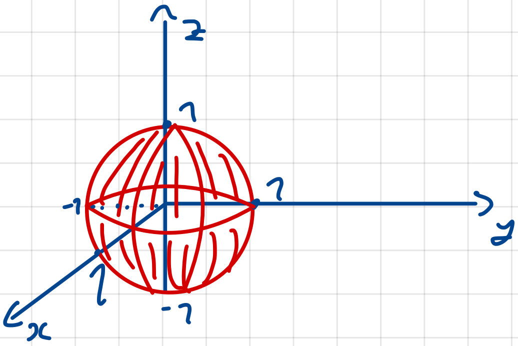


$\text{int}(D)=D \longrightarrow D$ é aberto

$\text{fr}(D)=\{(x,y)\in\mathbb{R}^2: x=0 \vee y=0\}$

f) $D=\{(x,y,z)\in\mathbb{R}^3: x^2+y^2+z^2\le 1\}$

$\hookrightarrow$ esfera de raio 1 e centro na origem

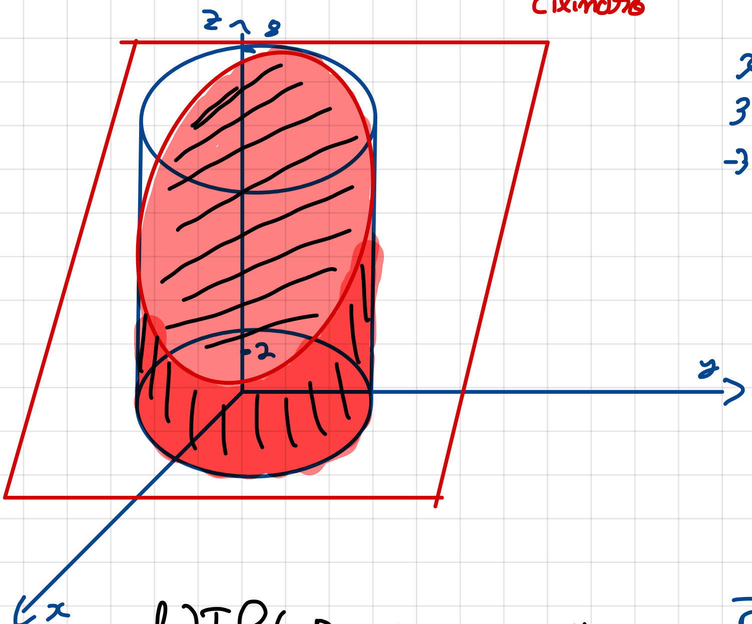


$\text{int}(D)=\{(x,y,z)\in\mathbb{R}^3: x^2+y^2+z^2<1\}$

$\text{fr}(D)=\{(x,y,z)\in\mathbb{R}^3: x^2+y^2+z^2=1\}$

$\overline{D}=D \rightarrow D$ é fechado

g) $D=\{(x,y,z)\in\mathbb{R}^3: \underbrace{x^2+y^2\le 9}_{\text{cilindro}} \wedge z\ge 0 \wedge \underbrace{z\le 5-x}_{\text{plano}}\}$



x	$z=5-x$
3	2
-3	8

$\text{int}(D)=\{(x,y,z)\in\mathbb{R}^3: x^2+y^2<9 \wedge z>0 \wedge z<5-x\}$

$\text{fr}(D)=\{(x,y,z)\in\mathbb{R}^3: (x^2+y^2=9 \wedge z\ge 0 \wedge z\le 5-x) \vee \underbrace{(z=0 \wedge x^2+y^2\le 9)}_{\text{base inferior}} \vee \underbrace{(z=5-x \wedge x^2+y^2=9)}_{\text{base superior}}\}$

$\overline{D}=D \rightarrow D$ é fechado

h) T.P.C. $\rightarrow$ região compreendida entre 2 elipsoides

Não é aberto nem fechado